NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

SECOND INTERNATIONAL SELECTION TEST

Trinity College, Cambridge, 8th April 1990

Time allowed: Three-and-a-half hours.

1. Let $f(n)$ be a function defined on the set of positive integers and having its values in the same set. Suppose that $f(f(m) + f(n)) = m + n$ for all positive integers m, n. Find all possible values of $f(1990)$.

2. The squares of an $n \times n$ chessboard ($n \geq 2$) are labelled $1, 2, \dots, n^2$ in some order with every number occurring. Prove that there exist two neighbouring squares (ie. with a common edge) whose labels differ by at least n.

3. Given a set of points P in the xy-plane, we define a set P^* according to the following rule:

   $(x^*, y^*) \in P^*$ if and only if $xx^* + yy^* \leq 1$ for all $(x, y) \in P$.

   Find all triangles T such that T^* is obtained from T by a half-turn about the origin.