

# NST4

27.5.15

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1. In trapezoid  $ABCD$ , the sum of the lengths of the bases  $AB$  and  $CD$  is equal to the length of the diagonal  $BD$ . Let  $M$  be the midpoint of  $BC$ , and  $E$  the reflection of  $C$  in line  $DM$ . Prove that  $\angle AEB = \angle ACD$ .
2. A confederation has five states, each of which has exactly  $5n$  airports. There are five airlines, which only operate interstate flights, such that every two airports in different states are connected by a direct (two-way) flight operated by exactly one of these airlines. Determine the greatest integer  $D$  satisfying the following condition: In every such confederation, it is possible to choose one of the five airlines and  $D$  of the  $25n$  airports such that one may travel (not necessarily directly) from any one of the  $D$  chosen airports to any other of the chosen airports only using flights operated by the chosen airline.
3. Determine all functions  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  satisfying

$$f(f(m) + n) + f(m) = f(n) + f(3m) + 2014$$

for all integers  $m$  and  $n$ .