NST 4, Eton and Oundle, 2009

A → C

a, not a+1

1. The triangle ABC has a right angle at C. Let P be a point inside ABC such that $|AP| = |AC|$. Let M be the midpoint of the hypotenuse AB and T be the foot of the altitude dropped from X. Prove that PM is a bisector of $\angle BPT$ if, and only if, $\angle A = 60°$.

2. Let $n > 1$ be a positive integer. Find all n-tuples $(a_1, a_2, \ldots, a_n)$ of positive integers which are pairwise distinct, pairwise coprime, and such that for each i in the range $1 \le i \le n$ we have $a+1 + a_2 + \cdots + a_n$ divides $a_1^i + a_2^i + \cdots + a_n^i$.

3. A graph has 15 vertices. There is at most one edge between each pair of vertices, and no edge connects a vertex to itself. Each edge is coloured either red or blue so that there are no monochromatic triangles. Find the largest possible number of edges of this graph.