NST 2

Tonbridge School, 25 May 2015

1. Let $n \geq 2$ be an integer, and let A_n be the set
$$A_n = \{2^n - 2^k \mid k \in \mathbb{Z}, 0 \leq k < n\}.$$
Determine the largest positive integer that cannot be written as the sum of one or more (not necessarily different) elements of A_n.

2. Let ABC be a triangle. The points K, L and M lie on the segments BC, CA and AB respectively, and the three lines AK, BL and CM are concurrent. Prove that it is possible to choose two of the triangles ALM, BMK and CKL such that their inradii sum to at least the inradius of ABC.

3. Let M be a set of $n \geq 4$ points in the plane, no three of which are collinear. Initially these points are connected with n line segments so that each point is the endpoint of exactly two segments. Then, at each step, one may chose two segments AB and CD sharing a common interior point, and replace them by two segments AC and BD provided that neither is present prior to the replacement. Prove that it is impossible to perform $n^3/4$ such moves.