

# Next Selection Test: Exam 2

IMO camp, Oundle School

26-v-2008

**Problem 4** Let  $n > 1$  be a positive integer and let  $X$  be a set of size  $n$  containing subsets  $A_1, A_2, \dots, A_{101}$  such that the union of any 50 of them has more than  $50n/51$  elements. Prove that among these given subsets there are three such that each pair of the three have non-empty intersection.

**Problem 5** Find the set of positive integers of the form

$$\frac{a^2 + ab + b^2}{ab - 1}$$

where  $a$  and  $b$  are non-negative integers and  $ab \neq 1$ .

**Problem 6** Let  $n$  be a positive integer. Suppose that  $n$  distinct coplanar unit vectors are positioned so that their tails are located at  $O$ . Suppose further that for some non-negative integer  $m < n/2$ , each closed half-plane with boundary containing  $O$  has the property that it contains at least  $m$  of the unit vectors under discussion. Show that the length of the sum of the vectors is at most  $n - 2m$ .

Time allowed: 4 hours 30 minutes