Next Selection Test: 4 hours 30 minutes

Oundle, May 28, 2003

1. Let n be a positive integer. A sequence of n positive integers (not necessarily distinct) is called *full* if it satisfies the following condition: for each positive integer $k \geq 2$, if the number k appears in the sequence then so does $k-1$, and moreover the first occurrence of $k-1$ comes before the last occurrence of k. For each n, how many full sequences are there?

2. Is there a positive integer m such that the equation
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{abc} = \frac{m}{a+b+c}$$
has infinitely many solutions in positive integers a, b, c?

3. For any set S of five points in the plane, no three of which are colinear, let $M(S)$ and $m(S)$ denote the greatest and smallest areas, respectively, of triangles determined by three points from S. What is the minimum possible value of $M(S)/m(S)$?