Next Selection Test: 4 hours 30 minutes

Oundle, June 5, 2002

1. Let ABC be a triangle and l the line through C which is parallel to AB. The internal bisector of angle A meets the side BC at D and the line l at E. The internal bisector of angle B meets the side AC at F and the line l at G. Suppose that $GF = DE$. Show that $AC = BC$.

2. Let $a_1, a_2, \ldots, a_n$ be non-negative real numbers, not all 0.

   (a) Show that $x^n - a_1x^{n-1} - a_2x^{n-2} - \cdots - a_{n-1}x - a_n$ has exactly one positive root.

   (b) Let $A = \sum_{j=1}^{n} a_j$ and $B = \sum_{j=1}^{n} ja_j$ and R be the positive root given by part (a). Show that $A^A \leq R^B$.

3. (a) Let n be a positive integer. Show that there exist distinct positive integers x, y, z such that

$$x^{n-1} + y^n = z^{n+1}.$$

   (b) Let a, b, c be positive integers such that a and b are relatively prime, and c is relatively prime either to a or b. Show that there are an infinity of positive integer triples (x, y, z) such that

$$x^a + y^b = z^c.$$