

First Selection Test: Exam 1

IMO camp, Trinity College Cambridge

5-iv-2008

Problem 1 A triangle ABC is given. A circle Γ passes through A and is tangent to the side BC at a point P . The circle Γ intersects AB and AC at points M and N respectively. Prove that the (minor) arcs MP and NP are equal if and only if Γ is tangent to the circumcircle of ABC at A .

Problem 2 In a certain country there are 10 cities connected by a network of one-way non-stop flights so that it is possible to fly (using one or more flights) from any city to any other. Let n be the least number of flights needed to complete a trip starting from one of the cities, visiting all the others, and returning to the starting point. Find the greatest possible value of n .

Problem 3 For each non-negative integer n , determine two integers p_n and q_n with the following property: there are exactly n non-negative integers x such that $x^2 + p_nx + q_n$ is a perfect square.

Time allowed: 4 hours 30 minutes